and that mathematics are no mere dull study—no mere tools for the man of science and the engineer but a world in themselves, demanding the exercise of insight, energy and hard work, by him who would possess its pure delights. That they are a branch of Nature study going deeper into the intricacies of that world of which we form a part, than the easier and more quickly acquired branches, a branch in which there is much work yet to be done, and that Nature says to us with regard to this branch to—

> Come wander away with me
> Into regions yet untrod,
> And read what is still unread
> In the manuscripts of God.

THE STUDY OF ELEMENTARY MATHEMATICS.

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However we may try to define education we know that the aim throughout is to develop, to turn the potentialities (if I may use such a word) of the child into actualities, to make him a useful member of society in the fullest sense. Our school curriculum, then our presentation of it, then our method of proceeding is settled with that object.

My work, this morning, is to indicate very shortly what we hope to do with mathematics, why we put it into the curriculum, and what we are going to do with it there; and let me make it clear at the outset that I fear I have nothing original to say, and to acknowledge my wholesale indebtedness to Mr. Branford.

We certainly do not include algebra and geometry because our boys and girls will, in after life, have to solve quadratic equations or treat with similar figures. We include mathematics, we look upon them as a very important part of the curriculum on account of the training they may give, and ought to give, to head and hand, and the discipline which forms character as Mrs. Hickson has already shown us.

That mathematics can give such training to the normal child is recognized. I do not want to open a discussion as to whether the mathematician has a clear quick grasp of matters and perhaps, of men, is in fact clear-headed, because he is a mathematician, or whether he is a mathematician because nature has endowed him with a clear head. Doubtless it would be most interesting. I am thinking at present about the help that mathematical training can be to the ordinary girl or boy. One finds curious acknowledgments of that help.

I fancy parents may be ready to admit the necessity of mathematics, and shall I say to acquiesce in the decision of those who settle that the subject must have a definite place in the curriculum, but they forget that they have their part to play. The attitude at home to the so-called mystery is often the cause of the rebuff administered to the teacher, who is doing all he can to make all the class develop their powers, when the remark is made: "Oh, I shall never be any good at mathematics, it is not in my family." Whence does this interesting fact come? Very few families can claim an Isaac Newton or a Lord Kelvin; teachers of elementary mathematics do not expect to find geniuses, they are trying to use elementary mathematics as training in thought, in expression of that thought, or shall we say in accurate thinking and in accuracy of the spoken or written result of that thought.

Part of the dislike and of the extra awe with which even the word mathematics is mentioned by a good many people is due to the fact that our forerunners—I am speaking as a teacher—looked at mathematics from a mathematician's point of view, or perhaps thought too much of the discipline and drill of the subject and forgot to use it as training in individual thought. Arithmetic was looked upon as a collection of rules and a great deal of mechanical application of the same to artificial or highly technical sums; algebra was a wonderful science of *a*'s and *x*'s; while in geometry perhaps the biggest mistake of all was made. Euclid, the logician, set himself the task of proving the well-known geometrical facts from a few axioms, the result was a *tour-de-force* providing excellent ground for discussion by the trained mind, but absolutely unsuited to children who, however, were made to study it.

Of course, in all ages there have been born teachers who have used algebra and even Euclid to the fullest advantage, but they are rare. Most have taught, and their pupils have handed the method straight on, one new proposition at a time, a page of factors all on the same model one day and the next rule the next, and so on. Lately we have been awakening to the opportunities we were missing, and to some of the mistakes we were making, and we are trying to improve matters, but we must be careful to take due consideration of the aim we have in view.

The potential training in mathematics for head and hand, for clear reasoning and for clear expression of that reasoning both by word of mouth and on paper is acknowledged. How are we to set to work ?

First, I think, we must remember that the history of the individual proceeds on the same lines as the history of the race, that as knowledge has been acquired by the race so it must be acquired by the individual; some of the steps may be hastened, may almost appear to be lost, but they are there. Attention to this fact may help us in settling the order and the method of the presentation of certain subjects to children; for example, our system of notation—the "nought" difficulty is made distinctly easier if we go through some steps equivalent to the abacus, then the graphical abacus, and lastly come to our notation.

Mathematical knowledge has arisen in response to certain impulses; the practical impulse, the scientific impulse and the æsthetic impulse, and they have acted in that order. There is hardly any branch of mathematics which did not begin from the necessity of solving a practical difficulty, the systematizing of the results, and the continuation of the subject from pure interest in mathematics, possibly from the idea of showing the unity of all mathematical knowledge; for example, trigonometry arose from the necessity of calculating three parts of a triangle when the other three are known. Then the trigonometrical ratios were formed into a system, and, lastly, the whole science was extended and is not by any means finished.

These impulses must be appealed to, and must be appealed to in that order. Any system of attacking arithmetic, algebra, or geometry, can and should appeal to the practical impulse first. Problems, I mean of course in the widest sense, can be suggested or can almost suggest themselves as requiring solution, and then that method of solution can be generalized. Possibly the impulse to study mathematics for its own sake has little effect on the normal child, but it does exist.

These are the internal stimuli; what about the external? First, I suppose, physical environment has played its part; we have used our fingers to suggest a scale of notation, we have used the measurements of parts of the human body as units—inch, cubit, foot, all arose in this way. Then occupations have from almost prehistoric times demanded such instruments as the abacus, which I mentioned just now, and the sundial, etc. The Nile removed the landmarks at its yearly rise, a means of recording this position was required, hence the beginning of all geometry, at least according to Herodotus. Then those sciences which are really applied mathematics, such as mechanics, astronomy, physics, demanded help. Lastly, chemistry, biology, sociology, and other branches of science required mathematical knowledge. Hardly a book is written on an economic or allied subject nowadays which is not illustrated by a graph, possibly only a simple one but often of a more elaborate nature.

Mathematics as a science has developed, has answered to these stumuli. What kinds of evidence or proof have satisfied the minds of the human race of the truth of mathematical propositions as the ages have passed along? First, experimental evidence or proof, then intuitional evidence or proof, lastly, scientific evidence or proof. Experimental evidence can only establish particular truths and suggest general truths, the mental activity called into play is chiefly sense-perception. Intuitional evidence or proof establishes the general truth and suggests the scientific ideal. Experiment and intuition are the means our pupils must use if they are to find the way to scientific systematization of the truths they have already acquired. This is the last stage. Let us take as an example the well-known and extremely useful theorem that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the sides containing the right angle. We feel sure that the Egyptians, 4,000 years ago, knew that a triangle the sides of which were in the ratio of 3 : 4 : 5 was right-angled; they knew it by experiment, and they also appear to have known that the more nearly the sides were in this ratio the more nearly was the largest angle a right angle. Then the Greeks knew by a dissection method that the fact was true for every right-angled triangle, and had what might be called a scientific proof for the isosceles right-angled triangle. Finally, Euclid enunciated the scientific proof with the familiar figure. Possibly we ought to follow steps very similar to these in introducing children to that particular truth, I do not mean in consecutive lessons. The first time they come across the truth it should be found out by experiment, by, so to speak, actually counting the squares, then later should come a dissection proof, and finally the scientific proof in its proper place.

I have suggested that the progress of mathematical knowledge in the individual must be similar to that which has been made by the race, then have considered the impulses or internal stimuli which have led to the desire for that knowledge, then the external stimuli, and lastly, the types of evidence which the mind can accept. To what guiding principles can these suggestions lead us?

First, I should quote the following :—

"(1) The particular mathematical experience which forms the material of the educational process must, at every stage, both in quantity and quality, be appropriate to the present capacity of the individual who is expected to assimilate it.

"(2) The correlation between the different branches of pure mathematics themselves, and between these latter and the manifold applications of mathematics must be natural, closely interdependent and continuous throughout."

Most of us know what mental indigestion is, and how often children will do a thing, will, say, divide or multiply because the teacher has done so, just as they will bolt their food if they are allowed to. We must see that we are not trying to force the logic of a mature mind down the throats of children who are still saying *why* in the sense of *how*. As regards correlation it must be natural and it must be true. We are *not* correlating geography and arithmetic if we set a sum about the basin of the Severn, the annual rainfall, and the rate of flowing to the sea, etc. We are correlating geometry and geography if we see that the properties of similar triangles help towards making maps correctly, or indeed, towards making maps at all. We are correlating geometry and handwork if we use our geometrical constructions for making patterns and designs, or applying mathematical properties to make mechanical toys.

When we have decided what we shall give our pupils at various ages, how shall we present it?

First, never take away from a child "its sacred right of discovery." Use it to the full in mathematics. Children can make their own multiplication table, they can discover the mechanical short cuts, they can discover geometrical truths, particularly they can make their own definitions. A definition, however crude, should be accepted if it covers all that the pupil can be expected to grasp at that stage.

Second, never introduce a symbol or contraction, even the simplest, except to satisfy a felt need. How much harm has been done by writing indices before they were demanded. I myself have often found that in slightly more advanced work by repeating the phrase "the index of the power to which *a* must be raised to give *x*," I have created a need for an abbreviation and have satisfied it by the word logarithm. Contractions are conventions, and when it is evident that a symbol is needed, the conventional one should be supplied.

Lastly, only that extent of the syllabus should be attempted which the pupil can (1) rationally grasp the basis of, (2) attain mechanical dexterity in. If this is to be done each child must be allowed, as far as possible, to go its own pace. The average pupil will not be any better off for knowing something about progressions if he does not understand the so-called rule of signs. Better to sacrifice the teaching of the class as a whole than to let one child be just copying what has been done on the board.

We want to teach each child to work, and to work himself, and to produce satisfactory accurate results. We have, perhaps, in former times stamped upon originality of work because we were so anxious to acquire accuracy, but it is very sad to be told as I was the other day, "Oh yes, the boys enjoy their work much more, things are much pleasanter, but in the old days when a boy was to be taken on in some business he was given a long sum to do, and he got it right, now he gets it wrong."

We want by our reform in the teaching or presentation of elementary mathematics to develop the individual, to give him his chance of self-realization, but we must be careful at the same time to see that he is being trained in accuracy, and that his interest in his sum extends to an interest in seeing the result is correct.